Calculus from Scratch: Complete Formula Guide & Practice Worksheet

Designed for CBSE Class 11 & 12 Students from a Non-Mathematics Background

Essential Mathematical Tools for Physics (PCB), Physical Chemistry, Economics & Applied Sciences

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How to Use This Guide: If you chose Biology (PCB), Commerce, or Humanities without Core Mathematics, encountering $\frac{dy}{dx}$ and $\int y\,dx$ in Physics, Chemistry, or Economics can feel overwhelming. This guide strips away heavy theoretical proofs and teaches you **what calculus actually means**, **every formula you need**, **how to avoid common algebra traps**, and **35 graded practice problems with full step-by-step solutions**.

1. The Big Picture: What Are Differentiation and Integration?

Calculus is built on just two opposite ideas that describe how quantities change and accumulate:

Feature	Differentiation ($\frac{dy}{dx}$)	Integration ($\int y\,dx$)
Core Idea	**Splitting up** to find the *instantaneous rate of change* of y with respect to x.	**Adding up** tiny pieces ($y \cdot dx$) to find the *total accumulation*.
Geometric Meaning	Gives the **slope of the tangent line** ($\tan\theta$) to the curve $y = f(x)$ at any point.	Gives the **area under the curve** $y = f(x)$ between two limits $x = a$ and $x = b$.
Physics Example	Finding instantaneous velocity from position: $v = \dfrac{dx}{dt}$.	Finding total displacement from velocity: $\Delta x = \displaystyle\int_{t_1}^{t_2} v\,dt$.
Economics Example	Marginal Cost from Total Cost: $MC = \dfrac{d(TC)}{dq}$.	Total Cost from Marginal Cost: $TC = \displaystyle\int MC\,dq + FC$.

Key Mental Model: Think of Differentiation and Integration as inverse operations—just like multiplication and division. Differentiating x^2 gives $2x$; integrating $2x$ brings you back to x^2 (plus a constant C).

2. Pre-Calculus Algebra & Trigonometry Survival Kit

Before using any calculus formula, **90% of mistakes made by non-Math students happen while rewriting roots and fractions**. Always convert roots and denominators into simple powers (x^n) *before* differentiating or integrating.

2.1 Essential Exponent & Logarithm Conversions

Expression	Rewrite As (x^n)	Logarithm Rule	Formula
$\frac{1}{x}$	x^{-1}	Product Rule	$\ln(a \cdot b) = \ln a + \ln b$
$\frac{1}{x^n}$	x^{-n}	Quotient Rule	$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
$\sqrt{x}$	$x^{1/2}$	Power Rule	$\ln(a^n) = n \ln a$
$\frac{1}{\sqrt{x}}$	$x^{-1/2}$	Base Values	$\ln e = 1, \quad \ln 1 = 0$
$\sqrt[3]{x^2}$	$x^{2/3}$	Base Change (Chem)	$\log_{10} x = \frac{\ln x}{2.303}$

2.2 Essential Trigonometry for Calculus

In all calculus formulas, angles MUST be measured in **radians** (π rad $= 180°$, so $30° = \frac{\pi}{6}$, $45° = \frac{\pi}{4}$, $60° = \frac{\pi}{3}$, $90° = \frac{\pi}{2}$).

- **Pythagorean Identities:** $\sin^2 x + \cos^2 x = 1$ and $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$.
- **Double-Angle Formulas (Crucial for Integration):** $\sin(2x) = 2 \sin x \cos x$, $\cos^2 x = \frac{1 + \cos(2x)}{2}$, $\sin^2 x = \frac{1 - \cos(2x)}{2}$.
- **Small-Angle Approximation (For very small θ in radians, $\theta < 5°$):** $\sin\theta \approx \theta$, $\tan\theta \approx \theta$, $\cos\theta \approx 1$.

3. Complete Differentiation Formula Sheet

3.1 Fundamental Rules of Differentiation

Rule Name	Formula	Quick Example
1. Constant Rule	$\frac{d}{dx}(c) = 0$ (where c is any standalone constant)	$\frac{d}{dx}(7) = 0, \quad \frac{d}{dx}(\pi^2) = 0$
2. Power Rule	$\frac{d}{dx}(x^n) = nx^{n-1}$ (works for any real number n)	$\frac{d}{dx}(x^4) = 4x^3$
3. Constant Multiple	$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$	$\frac{d}{dx}(5x^3) = 5(3x^2) = 15x^2$
4. Sum / Difference	$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$	$\frac{d}{dx}(x^2 + 3x) = 2x + 3$
5. Product Rule	$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$ (*First × deriv. of Second + Second × deriv. of First*)	$\frac{d}{dx}(xe^x) = xe^x + e^x(1)$
6. Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (*Low · d(High) − High · d(Low) over Low²*)	$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x}{x^2}$
7. Chain Rule	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (*Differentiate Outside, keep Inside same, × deriv. of Inside*)	$\frac{d}{dx}[\sin(3x)] = \cos(3x) \cdot 3$

3.2 Standard Derivatives Reference Table

Function $y = f(x)$	**Derivative** $\frac{dy}{dx}$	**Function** $y = f(x)$	**Derivative** $\frac{dy}{dx}$
x	1	$\sin x$	$\cos x$
$\sqrt{x} = x^{1/2}$	$\frac{1}{2\sqrt{x}}$	$\cos x$	$-\sin x$
$\frac{1}{x} = x^{-1}$	$-\frac{1}{x^2}$	$\tan x$	$\sec^2 x$
e^x	e^x	$\cot x$	$-\csc^2 x$
e^{ax}	ae^{ax}	$\sec x$	$\sec x \tan x$
$a^x \quad (a > 0)$	$a^x \ln a$	$\csc x$	$-\csc x \cot x$
$\ln x = \log_e x$	$\frac{1}{x}$	$\sin(ax + b)$	$a\cos(ax + b)$
$\log_a x$	$\frac{1}{x \ln a}$	$\cos(ax + b)$	$-a\sin(ax + b)$

Memory Trick for Trig Signs: Every trigonometric function starting with the letter **"C"** ($\cos x$, $\cot x$, $\csc x$) has a **negative sign** $(-)$ in its derivative!

4. Applications of Derivatives: Maxima, Minima & Rates

4.1 How to Find Maximum or Minimum Value of $y = f(x)$

Whether you want to find the maximum height of a projectile, maximum power in a circuit, minimum cost, or maximum profit, follow this exact 3-step recipe:

1. **Step 1 (Find Stationary Points):** Differentiate y to get $\frac{dy}{dx}$ and set $\frac{dy}{dx} = 0$ (because at the peak or valley of a smooth curve, the tangent is horizontal and its slope is zero). Solve for x.
2. **Step 2 (Find Second Derivative):** Differentiate $\frac{dy}{dx}$ once more to get $\frac{d^2y}{dx^2}$.
3. **Step 3 (Check Sign of $\frac{d^2y}{dx^2}$ at the critical point $x = x_0$):**
   - If $\frac{d^2y}{dx^2} < 0$ (**negative**), then $x = x_0$ gives a **Local Maximum**.
   - If $\frac{d^2y}{dx^2} > 0$ (**positive**), then $x = x_0$ gives a **Local Minimum**.

4.2 Stream-Specific Derivative Formulas (Physics & Economics)

Subject Area	Physical / Economic Quantity	Calculus Formula
Physics (Kinematics)	Instantaneous Velocity from Position $x(t)$	$v = \dfrac{dx}{dt}$
Physics (Kinematics)	Instantaneous Acceleration from Velocity $v(t)$ or $v(x)$	$a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = v\dfrac{dv}{dx}$
Physics (Mechanics)	Force from Linear Momentum $p(t)$ or Potential Energy $U(x)$	$F = \dfrac{dp}{dt}, \quad F = -\dfrac{dU}{dx}$
Physics (Electricity)	Electric Current from Charge $q(t)$; EMF from Magnetic Flux $\Phi_B(t)$	$I = \dfrac{dq}{dt}, \quad \varepsilon = -\dfrac{d\Phi_B}{dt}$
Chemistry (Kinetics)	Instantaneous Rate of Reaction for $R \to P$	$\text{Rate} = -\dfrac{d[R]}{dt} = +\dfrac{d[P]}{dt}$
Economics / Commerce	Marginal Cost (MC), Marginal Revenue (MR), Profit ($P = TR - TC$)	$MC = \dfrac{d(TC)}{dq}, \quad MR = \dfrac{d(TR)}{dq}$

5. Complete Integration Formula Sheet

5.1 Indefinite Integration Formulas ($\int f(x)\,dx = F(x) + C$)

Why $+C$? Because the derivative of any constant is 0 (e.g., $\frac{d}{dx}(x^2+5) = 2x$ and $\frac{d}{dx}(x^2-9) = 2x$), reversing differentiation requires adding an arbitrary **constant of integration** C.

Integrand	Indefinite Integral	Integrand	Indefinite Integral
$\int k\,dx$ (constant k)	$kx + C$	$\int \sin x\,dx$	$-\cos x + C$
$\int x^n\,dx \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$	$\int \cos x\,dx$	$\sin x + C$
$\int \dfrac{1}{x}\,dx = \int x^{-1}\,dx$	$\ln\lvert x\rvert + C$	$\int \sec^2 x\,dx$	$\tan x + C$
$\int e^x\,dx$	$e^x + C$	$\int \csc^2 x\,dx$	$-\cot x + C$
$\int e^{ax+b}\,dx$	$\dfrac{e^{ax+b}}{a} + C$	$\int \sin(ax+b)\,dx$	$-\dfrac{\cos(ax+b)}{a} + C$
$\int (ax+b)^n\,dx \quad (n \neq -1)$	$\dfrac{(ax+b)^{n+1}}{a(n+1)} + C$	$\int \cos(ax+b)\,dx$	$\dfrac{\sin(ax+b)}{a} + C$

Crucial Rule for $(ax+b)$ Inside an Integral: Whenever x is replaced by a linear expression $(ax+b)$, integrate using the normal formula and **divide the result by the coefficient of x (which is a)!**

5.2 Definite Integration ($\int_a^b f(x)\,dx$) & Physics Applications

In a **Definite Integral**, we integrate between a lower limit $x = a$ and an upper limit $x = b$. The constant C cancels out:

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a) \qquad \text{(Upper Limit Value} - \text{Lower Limit Value)}$$

- **Displacement from Velocity:** $\Delta x = x_2 - x_1 = \int_{t_1}^{t_2} v(t)\,dt$ **Change in Velocity:** $\Delta v = v_2 - v_1 = \int_{t_1}^{t_2} a(t)\,dt$
- **Work Done by a Variable Force:** $W = \int_{x_1}^{x_2} F(x)\,dx$ **Charge Flown in Time Interval:** $\Delta q = \int_{t_1}^{t_2} I(t)\,dt$

6. Step-by-Step Worked Examples (Learn the Technique)

Example 1 (Handling Roots & Fractions before Differentiating): Differentiate $y = 4x^3 - \dfrac{6}{x^2} + 8\sqrt{x} - 9$ with respect to x.

- **Step 1 (Rewrite everything as x^n):** $y = 4x^3 - 6x^{-2} + 8x^{1/2} - 9$.
- **Step 2 (Apply Power Rule $\frac{d}{dx}(x^n) = nx^{n-1}$ term by term):**

$$\frac{dy}{dx} = 4(3x^{3-1}) - 6(-2x^{-2-1}) + 8\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - 0 = 12x^2 + 12x^{-3} + 4x^{-1/2} = 12x^2 + \frac{12}{x^3} + \frac{4}{\sqrt{x}}.$$

Example 2 (The Chain Rule / "Outside-Inside" Rule): Find $\dfrac{dy}{dx}$ if $y = (3x^2 + 5)^4$.

- **Step 1 (Identify Outside & Inside):** Outside box is $(\text{box})^4$; Inside is $u = 3x^2 + 5$.
- **Step 2 (Differentiate Outside $\times$ Differentiate Inside):**

$$\frac{dy}{dx} = 4(3x^2 + 5)^{4-1} \cdot \frac{d}{dx}(3x^2 + 5) = 4(3x^2 + 5)^3 \cdot (6x) = 24x(3x^2 + 5)^3.$$

Example 3 (Kinematics Application): The position of a particle moving along the x-axis is given by $x(t) = 2t^3 - 9t^2 + 12t + 4$ (where x is in metres and t in seconds). Find: (a) velocity at $t = 2$ s, (b) time(s) when the particle momentarily comes to rest, and (c) acceleration at $t = 2$ s.

- **(a) Velocity function:** $v(t) = \dfrac{dx}{dt} = \dfrac{d}{dt}(2t^3 - 9t^2 + 12t + 4) = 6t^2 - 18t + 12$. At $t = 2$ s: $v(2) = 6(4) - 18(2) + 12 = 0$ m/s.
- **(b) Particle at rest means $v(t) = 0$:** $6t^2 - 18t + 12 = 0 \implies t^2 - 3t + 2 = 0 \implies (t-1)(t-2) = 0 \implies t = 1$ s and $t = 2$ s.
- **(c) Acceleration function:** $a(t) = \dfrac{dv}{dt} = 12t - 18$. At $t = 2$ s: $a(2) = 12(2) - 18 = +6$ m/s^2.

Example 4 (Maxima & Minima): Find the maximum and minimum values of $y = x^3 - 6x^2 + 9x + 15$.

- **Step 1:** $\dfrac{dy}{dx} = 3x^2 - 12x + 9$. Setting $\dfrac{dy}{dx} = 0$: $3(x^2 - 4x + 3) = 0 \implies (x-1)(x-3) = 0 \implies x = 1$ or $x = 3$.
- **Step 2:** $\dfrac{d^2y}{dx^2} = 6x - 12$.
- **Step 3:** At $x = 1$: $\dfrac{d^2y}{dx^2} = 6(1) - 12 = -6 < 0 \implies$ **Maximum** at $x = 1$, with $y_{\max} = (1)^3 - 6(1)^2 + 9(1) + 15 = 19$.
  At $x = 3$: $\dfrac{d^2y}{dx^2} = 6(3) - 12 = +6 > 0 \implies$ **Minimum** at $x = 3$, with $y_{\min} = (3)^3 - 6(3)^2 + 9(3) + 15 = 15$.

Example 5 (Definite Integration & Work Done): A variable force $F(x) = (4x + 2)$ N acts on a body along the x-axis. Find the work done in moving the body from $x = 1$ m to $x = 3$ m.

- $W = \int_1^3 (4x+2)\,dx = \left[\frac{4x^2}{2} + 2x\right]_1^3 = \left[2x^2+2x\right]_1^3 = (2(3)^2+2(3))-(2(1)^2+2(1)) = (18+6)-(2+2) = 20$ J.

7. Graded Practice Worksheet (35 Problems)

Level 1: Warm-Up & Power Rule Mastery (Differentiate w.r.t. x)

1. $y = 5x^4 - 3x^2 + 11x - 8$
2. $y = x^7 + \frac{1}{x^3} - \frac{4}{x}$
3. $y = 6\sqrt{x} + \frac{2}{\sqrt{x}}$
4. $y = 3x^{5/3} - 4x^{-1/4} + \pi^3$
5. $y = \frac{x^4 - 5x^2 + 2}{x}$ *(Hint: Split into individual fractions before differentiating!)*
6. $y = (2x + 3)(x - 4)$ *(Hint: Expand first or use Product Rule.)*
7. $y = 4\sin x - 3\cos x + 2e^x$
8. $y = 5\ln x + 7\tan x - \frac{3}{x^2}$

Level 2: Product Rule, Quotient Rule & Chain Rule (Find $\frac{dy}{dx}$)

9. $y = x^2 \sin x$
10. $y = e^x \ln x$
11. $y = \frac{x+1}{x-1}$
12. $y = \frac{\sin x}{1+\cos x}$
13. $y = (4x^2 - 7x + 2)^5$
14. $y = \sqrt{1 + 3x^2}$
15. $y = \sin(5x + 2) + \cos(3x^2)$
16. $y = e^{-4x} + \ln(2x^3 + 1)$

Level 3: Indefinite & Definite Integration (Evaluate Each Integral)

17. $\int \left(6x^2 - 8x + 5\right) dx$
18. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} - \frac{3}{x^2}\right) dx$
19. $\int \left(\frac{4}{x} + 3e^x - 2\sin x\right) dx$
20. $\int (3x - 2)^4\,dx$
21. $\int \cos(4x + 1)\,dx$

22. $\displaystyle\int_0^2 \left(3x^2 + 2x + 1\right) dx$
23. $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}\, dx$
24. $\displaystyle\int_0^{\pi/2} \sin x\, dx$

Level 4: Rate of Change, Tangent Slope & Maxima/Minima

25. Find the slope of the tangent to the curve $y = 3x^2 - 4x + 7$ at the point where $x = 2$.
26. The radius r of a circular wave on a pond is increasing at a constant rate of $\dfrac{dr}{dt} = 3$ cm/s. How fast is the enclosed circular area $A = \pi r^2$ increasing when the radius is $r = 10$ cm? *(Hint: Use Chain Rule* $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$*.)*
27. Find the value of x for which the function $y = 2x^2 - 8x + 11$ reaches its minimum value, and find that minimum value.
28. Find both the local maximum and local minimum values of $y = 2x^3 - 15x^2 + 36x + 10$.
29. The sum of two positive numbers x and y is 20 (so $x + y = 20$). Using calculus, find the two numbers such that their product $P = x \cdot y$ is maximum.

Level 5: Real-World CBSE Applications (Physics, Chemistry & Economics)

30. **[Physics – Kinematics]** The displacement s (in metres) of a car at time t (in seconds) is given by $s(t) = 4t^2 + 10t + 6$. Find (a) its initial velocity at $t = 0$, (b) its velocity at $t = 3$ s, and (c) its constant acceleration.
31. **[Physics – Projectile / Vertical Motion]** A ball thrown vertically upward has height $h(t) = 20t - 5t^2$ (where h is in metres and t in seconds). Using differentiation, find (a) the time t when it reaches maximum height, and (b) the maximum height reached.
32. **[Physics – Electric Current & Charge]** The charge flowing through a cross-section of a wire varies with time as $q(t) = 3t^2 + 2t + 1$ coulombs. Find (a) the instantaneous current $I = \dfrac{dq}{dt}$ at $t = 2$ s, and (b) if instead a current $I(t) = 4t$ A flows from $t = 0$ s to $t = 3$ s, find the total charge $\Delta q = \displaystyle\int_0^3 I(t)\, dt$ passed.
33. **[Physics – Work by Variable Force]** A spring exerts a restoring force magnitude $F(x) = 100x$ N, where x is the extension in metres. Calculate the work done in stretching the spring from $x = 0.1$ m to $x = 0.3$ m.
34. **[Economics – Marginal Cost]** The Total Cost TC (in Rs.) of producing q units of a commodity is given by $TC(q) = 0.5q^2 + 15q + 200$. Find (a) the Marginal Cost function $MC(q)$, and (b) the Marginal Cost when $q = 10$ units.
35. **[Economics – Profit Maximization]** A firm's Total Revenue is $TR(q) = 100q - q^2$ and Total Cost is $TC(q) = 20q + 100$, where q is the output quantity. Find the output q that maximizes Profit $\Pi(q) = TR(q) - TC(q)$, and calculate the maximum profit.

8. Complete Step-by-Step Answer Key

Solutions to Level 1 (Warm-Up & Power Rule)

1. $\frac{dy}{dx} = 5(4x^3) - 3(2x) + 11(1) - 0 = \mathbf{20x^3 - 6x + 11}$.
2. Rewrite as $y = x^7 + x^{-3} - 4x^{-1}$. Then $\frac{dy}{dx} = 7x^6 - 3x^{-4} + 4x^{-2} = \mathbf{7x^6 - \frac{3}{x^4} + \frac{4}{x^2}}$.
3. Rewrite as $y = 6x^{1/2} + 2x^{-1/2}$. Then $\frac{dy}{dx} = 6\left(\frac{1}{2}x^{-1/2}\right) + 2\left(-\frac{1}{2}x^{-3/2}\right) = \mathbf{\frac{3}{\sqrt{x}} - \frac{1}{x^{3/2}}}$.
4. Since π^3 is a constant, its derivative is 0: $\frac{dy}{dx} = 3\left(\frac{5}{3}x^{2/3}\right) - 4\left(-\frac{1}{4}x^{-5/4}\right) + 0 = \mathbf{5x^{2/3} + x^{-5/4}}$.
5. Split fraction: $y = x^3 - 5x + 2x^{-1}$. Differentiating gives $\frac{dy}{dx} = \mathbf{3x^2 - 5 - \frac{2}{x^2}}$.
6. Expand: $y = 2x^2 - 5x - 12$. Differentiating gives $\frac{dy}{dx} = \mathbf{4x - 5}$.
7. $\frac{dy}{dx} = 4(\cos x) - 3(-\sin x) + 2e^x = \mathbf{4\cos x + 3\sin x + 2e^x}$.
8. Rewrite last term as $-3x^{-2}$: $\frac{dy}{dx} = \mathbf{\frac{5}{x} + 7\sec^2 x + \frac{6}{x^3}}$.

Solutions to Level 2 (Product, Quotient & Chain Rules)

9. Product Rule: $\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^2) = \mathbf{x^2\cos x + 2x\sin x}$.
10. Product Rule: $\frac{dy}{dx} = e^x\left(\frac{1}{x}\right) + (\ln x)e^x = \mathbf{e^x\left(\frac{1}{x} + \ln x\right)}$.
11. Quotient Rule: $\frac{dy}{dx} = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} = \mathbf{-\frac{2}{(x-1)^2}}$.
12. Quotient Rule: $\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \mathbf{\frac{1}{1+\cos x}}$.
13. Chain Rule: $\frac{dy}{dx} = 5(4x^2 - 7x + 2)^4 \cdot (8x - 7) = \mathbf{(40x - 35)(4x^2 - 7x + 2)^4}$.
14. Rewrite as $y = (1 + 3x^2)^{1/2}$. By Chain Rule: $\frac{dy}{dx} = \frac{1}{2}(1 + 3x^2)^{-1/2} \cdot (6x) = \mathbf{\frac{3x}{\sqrt{1 + 3x^2}}}$.
15. Chain Rule on each term: $\frac{dy}{dx} = \cos(5x + 2) \cdot 5 - \sin(3x^2) \cdot (6x) = \mathbf{5\cos(5x + 2) - 6x\sin(3x^2)}$.
16. Chain Rule: $\frac{dy}{dx} = e^{-4x}(-4) + \frac{1}{2x^3 + 1} \cdot (6x^2) = \mathbf{-4e^{-4x} + \frac{6x^2}{2x^3 + 1}}$.

Solutions to Level 3 (Indefinite & Definite Integration)

17. $\frac{6x^3}{3} - \frac{8x^2}{2} + 5x + C = \mathbf{2x^3 - 4x^2 + 5x + C}$.
18. Rewrite as $\int \left(x^{1/2} + x^{-1/2} - 3x^{-2}\right) dx = \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} - \frac{3x^{-1}}{-1} + C = \mathbf{\frac{2}{3}x^{3/2} + 2\sqrt{x} + \frac{3}{x} + C}$.
19. $\mathbf{4\ln|x| + 3e^x + 2\cos x + C}$.
20. Linear rule ($a = 3, n = 4$): $\frac{(3x-2)^5}{3 \times 5} + C = \mathbf{\frac{(3x-2)^5}{15} + C}$.
21. Linear rule ($a = 4$): $\mathbf{\frac{\sin(4x+1)}{4} + C}$.
22. $\left[x^3 + x^2 + x\right]_0^2 = (8 + 4 + 2) - (0) = \mathbf{14}$.

23. $\displaystyle\int_1^4 x^{-1/2}\,dx = \Big[2\sqrt{x}\Big]_1^4 = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = \mathbf{2}.$

24. $\Big[-\cos x\Big]_0^{\pi/2} = -\cos(\pi/2) - (-\cos 0) = 0 - (-1) = \mathbf{1}.$

Solutions to Level 4 (Rate of Change, Tangent Slope & Maxima/Minima)

25. $\dfrac{dy}{dx} = 6x - 4$. At $x = 2$, slope $m = 6(2) - 4 = \mathbf{8}$.

26. $\dfrac{dA}{dr} = 2\pi r$. By Chain Rule, $\dfrac{dA}{dt} = 2\pi r \cdot \dfrac{dr}{dt} = 2\pi(10)(3) = \mathbf{60}\pi$ cm$^\mathbf{2}$/s $\approx \mathbf{188.5}$ cm$^\mathbf{2}$/s.

27. $\dfrac{dy}{dx} = 4x - 8 = 0 \implies \mathbf{x = 2}$. Since $\dfrac{d^2y}{dx^2} = 4 > 0$, it is a minimum. Minimum value: $y(2) = 2(4) - 8(2) + 11 = \mathbf{3}$.

28. $\dfrac{dy}{dx} = 6x^2 - 30x + 36 = 6(x-2)(x-3) = 0 \implies x = 2, 3$. Second derivative: $\dfrac{d^2y}{dx^2} = 12x - 30$. At $x = 2$, $\dfrac{d^2y}{dx^2} = -6 < 0$ (**Max**: $y(2) = \mathbf{38}$). At $x = 3$, $\dfrac{d^2y}{dx^2} = +6 > 0$ (**Min**: $y(3) = \mathbf{37}$).

29. Substitute $y = 20 - x$ into $P = x(20 - x) = 20x - x^2$. Then $\dfrac{dP}{dx} = 20 - 2x = 0 \implies x = 10$. Since $\dfrac{d^2P}{dx^2} = -2 < 0$, product is maximum when $\mathbf{x = 10}$, $\mathbf{y = 10}$ ($P_{\max} = 100$).

Solutions to Level 5 (Real-World CBSE Applications)

30. $v(t) = \dfrac{ds}{dt} = 8t + 10$. (a) At $t = 0$: $v(0) = \mathbf{10}$ m/s. (b) At $t = 3$ s: $v(3) = 8(3) + 10 = \mathbf{34}$ m/s. (c) $a = \dfrac{dv}{dt} = \mathbf{8}$ m/s$^\mathbf{2}$.

31. (a) At maximum height, velocity $v = \dfrac{dh}{dt} = 20 - 10t = 0 \implies \mathbf{t = 2}$ s (and $\frac{d^2h}{dt^2} = -10 < 0$). (b) $h_{\max} = 20(2) - 5(2)^2 = \mathbf{20}$ m.

32. (a) $I(t) = \dfrac{dq}{dt} = 6t + 2$; at $t = 2$ s, $I(2) = 6(2) + 2 = \mathbf{14}$ A. (b) $\Delta q = \displaystyle\int_0^3 4t\,dt = \Big[2t^2\Big]_0^3 = 2(9) - 0 = \mathbf{18}$ C.

33. $W = \displaystyle\int_{0.1}^{0.3} 100x\,dx = \Big[50x^2\Big]_{0.1}^{0.3} = 50(0.09 - 0.01) = 50(0.08) = \mathbf{4}$ J.

34. (a) $MC(q) = \dfrac{d(TC)}{dq} = \mathbf{q + 15}$. (b) At $q = 10$: $MC(10) = 10 + 15 =$ Rs. $\mathbf{25}$.

35. Profit $\Pi(q) = (100q - q^2) - (20q + 100) = -q^2 + 80q - 100$. Setting $\dfrac{d\Pi}{dq} = -2q + 80 = 0 \implies \mathbf{q = 40}$ units ($\frac{d^2\Pi}{dq^2} = -2 < 0$). Maximum Profit: $\Pi(40) = -(40)^2 + 80(40) - 100 =$ Rs. $\mathbf{1500}$.